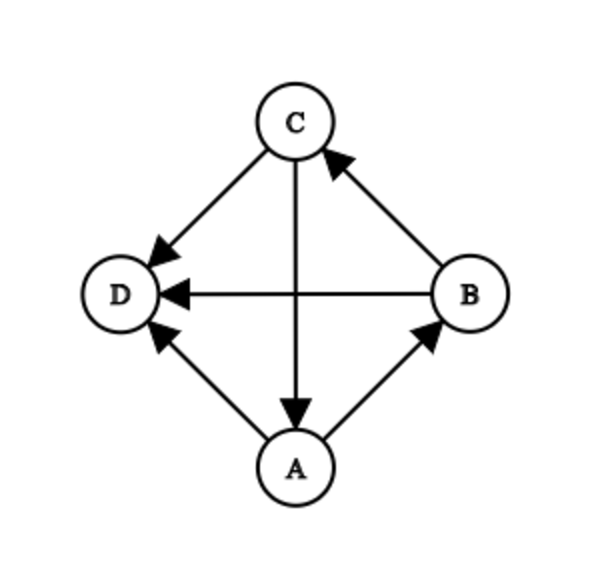
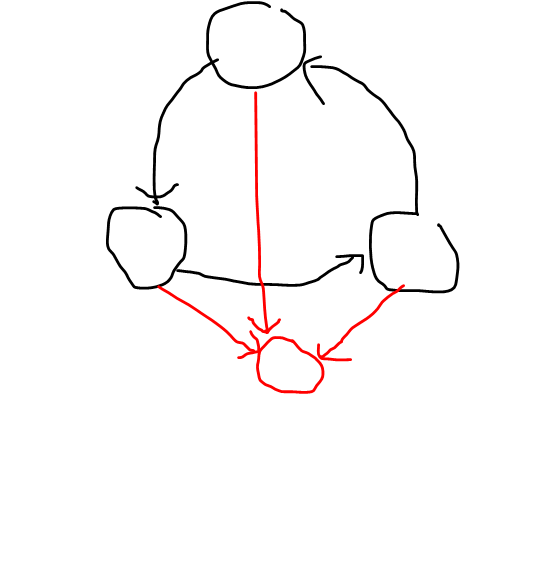
Given a complete directed graph with vertices not containing the below subgraph, find the sum of distances between all ordered pairs of vertices. ( if cannot go)



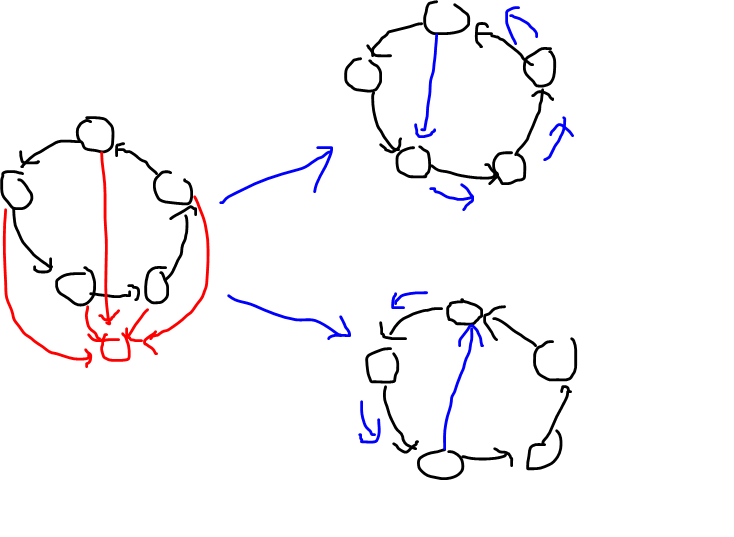
Observation 1:

The graph can be thought of as a cycle of length 3 all pointing to another vertex.

Observation 2:

Vertices with zero in-degree can be removed as their contribution can be calculated easily (bfs)

Define to be the set of nodes such that exists.



Observation 3:

does not contain cycles.

Explanation:

If there is a cycle bigger than 3, you can divide it into a smaller cycle no matter which direction an edge is (edges must all exist as it is a complete graph), and if you continue there will always be a cycle with size 3, which is the subgraph the graph don’t have, so it is impossible to have a cycle. (cycle can’t contain x either as both are pointing to x)

Let be a node with maximum in-degree. Let be and be .

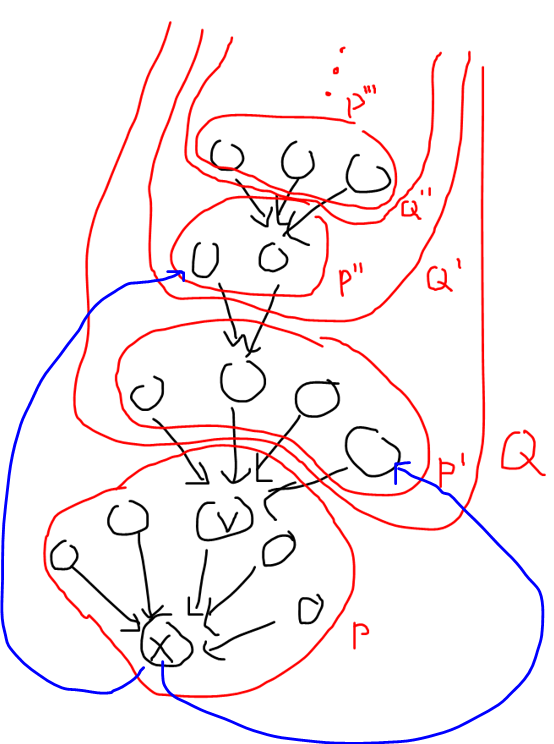
Observation 4: For all pairs , there is at least one pair such that exists.

Explanation: Since is acyclic, it can be topological sorted. This means there must be a vertex with no in-degree (within ). However, we removed all vertex with zero in-degree in the whole graph, so there must be some edge pointing to it that comes from .

Let be and be .

Observation 5: does not contain cycles.

Explanation: same as observation 3

Observation 6: does not contain cycles.

Explanation: You can repeat the above process until is broken up into a series of acyclic components. (Very cool!) Also cycles cannot form between components in as it is a bamboo graph. Edges from to vertices in can form cycles, but it doesn’t matter as the cycles are not completely in . (obviously will not connect to other vertices in )

Now we proved that the graph can be separated into 2 acyclic sets ( and ), we can topological sort them and do some stuff.

Label vertices in such that if , exists. (same for )

Observation 7:

if , you can go from to to .

to must exist as if it’s the opposite will be in

if ,